

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE Aug. 15, 1996	3. REPORT TYPE AND DATES COVERED Final Report: June 1, '93 -Aug.31, '96	
4. TITLE AND SUBTITLE  Developing new generation of models for turbulent flows			5. FUNDING NUMBERS  F49620- 93-1-0315	
6. AUTHOR(S)  K.R. Sreenivasan				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  Yale University 9 Hillhouse Ave., Mason Lab. New Haven, CT 06520-8286				
AFOSR-TR-96  0532				
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  AFOSR 110 Duncan Ave., Suite B115 Bolling Air Force Base Washington, DC 20332-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  F 49620-93-1-0315	
11. SUPPLEMENTARY NOTES  None				
12a. DISTRIBUTION/AVAILABILITY STA  unlimited			12b. DISTRIBUTION CODE  unlimited	
19961104 085				
13. ABSTRACT (Maximum 200 words)  The goal of the study was to generate, from a simple set of rules, a stochastic signal in space-time that would closely resemble the real turbulent data. This goal has been achieved and the work is reported in a journal article (Phys. Rev. E. 49, 5179, 1994) and Chapter 4.4 of the Ph.D thesis by A. Juneja (May 1995). The paper explains the principles and deals with one-dimensional signals whereas the thesis chapter deals with three-dimensional signals. These synthetic signals possess most of the statistical properties of real turbulence, and have been used as initial conditions in a direct numerical simulation of homogeneous turbulence. A comparable scheme was attempted for wall-bounded flows, and the principal hypothesis relating to this attempt was successfully verified in a high-Reynolds-number turbulent pipe flow and a moderate-Reynolds-number turbulent boundary layer.				
14. SUBJECT TERMS  synthetic turbulence, artificial turbulence, high-Reynolds-number turbulence			15. NUMBER OF PAGES 9 + cover	
			16. PRICE CODE ---	
17. SECURITY CLASSIFICATION OF REPORT unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT unclassified	20. LIMITATION OF ABSTRACT None	

# **Final report on the AFOSR AASERT Grant to Yale University, grant number F49620-93-1-0315**

submitted by

**K.R. Sreenivasan**  
Mechanical Engineering Department

## **Summary of accomplishments**

The goals of the research carried out under the AASERT Proposal were as follows:

(a) Generate from a simple set of rules data that are very close to the real turbulent data. These data could be used as initial conditions for direct numerical simulations. It was thought that the use of these data would enable faster convergence to the final state of turbulence than would be possible by starting with, say, a Gaussian random field as the initial data.

(b) Generate new subgrid models which faithfully account for the intermittency of the small scale.

As reported in previous progress reports, (a) and (b) were accomplished relatively quickly for homogeneous and isotropic turbulence. The basic ideas and their implementation in one dimension has been published [1]. The three-dimensional version of the model was used successfully to generate initial conditions for a simulation of turbulence in a periodic box. This part of the work is described in the Ph.D. thesis of Anurag Juneja ("Scaling laws in turbulence: their manifestation and utility", May 1995). This part of the work has not yet been published. In summary, the goals of the proposal can be said to have been met satisfactorily.

Even as the work just mentioned was getting under way, it was realized that a more interesting and useful task would be to extend the method to shear flows, especially wall-bounded flows such as boundary layers. A new scheme was devised to this end. The basic hypothesis underlying the scheme was verified experimentally. However, the method was not implemented and tested because the grant came to a natural close. This part of the work is being prepared for publication.

The publication and the thesis have been sent to AFOSR as part of previous progress reports, and the forthcoming publications will also be sent when ready.

## Some details

### Homogeneous turbulence

The basis of the work is the reasonably well-understood fact that the inertial and dissipative ranges of scales of turbulent motion at high Reynolds numbers are independent of the flow configuration---at least to a degree that is usefully accurate. It then follows that these scales of motion depend only on a few gross parameters. An important question, then, is: What is the smallest set of parameters which adequately incorporate all the universal aspects of turbulent motion---at least to some reasonable accuracy? One can construct, with relatively modest physical input, stochastic signals that have most properties of a turbulence velocity trace at high Reynolds numbers. For convenience, artificial signals mimicking real turbulence well were designated as synthetic turbulence.

One of the relevant observations in this regard is that the longitudinal velocity increments in the inertial range of scales share some of the properties of fractional Brownian motion with a Hurst exponent of about 0.35 [2]. However, this is only partially true because the increments of fractional Brownian motion are symmetrically distributed whereas the turbulent velocity increments have a finite skewness. Several other methods can produce signals possessing some properties of turbulence but fall short in other ways. A particular shortcoming of these models is that they do not successfully incorporate the skewness of the velocity derivative and odd-order structure functions. Indeed, the finite skewness of longitudinal velocity increments (or differences) is an important property related to the energy transfer across scales [3], and its incorporation into a simple scheme is a major challenge. Another important consideration is the incorporation of intermittency of the dissipation field.

In the present work, we have outlined a family of schemes for generating turbulence-like signals that mimic real turbulence; in the more refined of these schemes, the signals generated do not differ significantly from real turbulence in the sense of one-point and two-point statistics. All schemes consist of three essential ingredients. First, an appropriate multiplicative procedure is utilized for generating intermittent positive definite signals (or measures) possessing many of the properties of turbulent energy dissipation,  $\epsilon$  [4]. The properties of the measure living in a box of size  $r$  are then related

to those of velocity increments  $\Delta u(r)$  over a separation distance  $r$ . This is done *via* a stochastic variable  $V$  introduced in the spirit of Kolmogorov's refined similarity hypothesis [5]. The third and final step consists of constructing synthetic turbulent velocity signals by suitably combining the velocity increments. This step partially resembles the so-called mid-point displacement method [6] used for generating fractional Brownian motion.

For the three-dimensional extension of the method, the simplest scheme consists of generating three independent one-dimensional fields and enforcing the divergence-free condition on the total vector field. This straightforward procedure was found to be rather expensive computationally, and so some simplified procedures were invented. The cheapest of them involved only an approximate satisfaction of the periodic boundary conditions. Further work showed that the direct numerical simulations employing these synthetic data yielded better convergence than those with random initial conditions, as measured in terms of the decay of total vorticity and the derivative skewness and flatness. The DNS scheme was devised by G. Erlebacher of ICASE and the simulations were performed on a  $64^3$  and  $96^3$  boxes. Further details can be found in Juneja's thesis which was included as part of a previous progress report.

### Wall-bounded flows

The basic idea, just as in homogeneous turbulence, was to generate space-time velocity data in wall-bounded flows, such as pipe and channel flows as well as boundary layers. It seemed that the scheme would have the best chances of success if restricted to the so-called logarithmic region. In that region, it was hypothesized that one can relate, by means of a universal stochastic variable  $W$ , the dynamics of all velocity fluctuations in the constant-pressure boundary layer to that of a single dynamic variable, such as the (kinematic) wall shear stress,  $\tau_w$ . The average wall stress not only serves as the scale for average quantities such as mean velocity and the mean-square velocity fluctuation, but the instantaneous wall stress determines the properties of the instantaneous velocity fluctuation  $u$  in the flow, through the universal variable  $W$  which is independent of the distance from the wall (for  $y^+ > 30$ , say), the wall stress and the Reynolds number (as long as it is sufficiently high). This hypothesis, denoted here as the "strong similarity" hypothesis, can be stated as follows:

"In the overlap region of the boundary layer (also pipe flows, channel flows, and other

attached wall-bounded flows) one can define a stochastic variable  $W$  such that

$$u = W \tau_w^{1/2}.$$

The variable  $W$  is independent of the bulk Reynolds number of the flow (as long as it is sufficiently high) and of  $\tau_w$ , but depends on the height  $y$  from the wall. When  $y\tau_w^{1/2}/\nu \gg 1$ , the stochastic variable  $W$  becomes independent also of  $y$ , and is thus universal."

The work so far suggests that  $W$  is independent of the wall-normal distance to a good approximation. This is shown by Figure 1, which is the probability density of  $W$  at five different wall-normal positions at a Reynolds number of about 230,000 in a pipe flow. Note that the variable  $W$  is a dimensionless quantity and is not normalized by its standard deviation or referenced to anything else. The probability density is plotted in logarithmic coordinates to emphasize its tails. This figure encourages the belief that there is an extraordinary simplification in log-layer dynamics, and that, if one can provide a simple dynamical model for the wall shear stress, we can describe by means of that model and the variable  $W$ , the entire log-layer dynamics---at least to an accuracy that is suitable for most practical purposes.

We made similar measurements in the boundary layer at moderate Reynolds numbers. Figure 2 shows that  $W$  is essentially independent of the wall-normal position (at least in the narrow range of Reynolds number explored). For this flow as well, the function  $W$  was found to be independent of the wall-normal distance as expected (this will be seen in the next figure).

As stated earlier, the expectation was that the stochastic function  $W$  would be independent not only of the wall-normal distance but also of the flow (pipes, channels, wall jets and boundary layers). That is, the probability density of  $W$  would be universal. Comparison between boundary layers and the pipe flows shows that there is some difference. However, when normalized by the standard deviation of  $W$ , the pipe flow data at all the wall-normal positions and the boundary layer data at different heights from the wall collapse quite well. This is shown in Figure 3. Thus, it appears that a large part of the hypothesis is verified, but more work is needed to assess further details of the model and implement it for practical purposes.

## References

1. A. Juneja, D.P. Lathrop, K.R. Sreenivasan and G. Stolovitzky, Phys. Rev. E., 49, 5179 (1994).
2. K.R. Sreenivasan and A. Juneja, preprint, 1992.
3. A.S. Monin and A.M. Yaglom, *Statistical Fluid Mechanics*, volume 2, MIT Press (1975)
4. C. Meneveau and K.R. Sreenivasan, Phys. Rev. Lett. 59, 1424 (1987)
5. G. Stolovitzky and K.R. Sreenivasan, Phys. Rev. Lett., 69, 1178 (1992)
6. R.F. Voss, Physica D, 38, 362 (1989)

## Figure captions

Figure 1. The logarithm of the probability density of the variable  $W$  measured for five different heights in the log-region of a turbulent pipe flow at a Reynolds number of 230,000. Symbols correspond to  $y/R = 0.033, 0.067, 0.10, 0.133$  and  $0.167$ ,  $R$  being the pipe radius.

Figure 2. The logarithm of the probability density function of the variable  $W$  at a height of 3 mm in boundary layers at momentum thickness Reynolds numbers of 2147, 2666, 3900 and 4416. The 3 mm height lies within the log-region of all the boundary layers.

Figure 3. Comparison between pipe flows and boundary layers. The boundary layer positions are  $0.056\delta, 0.083\delta, 0.139\delta$ , where  $\delta$  is the boundary layer thickness; the pipe positions are  $0.033R, 0.067R$  and  $0.1R$ . When normalized by the standard deviation of  $W$ , all the plots collapse satisfactorily.

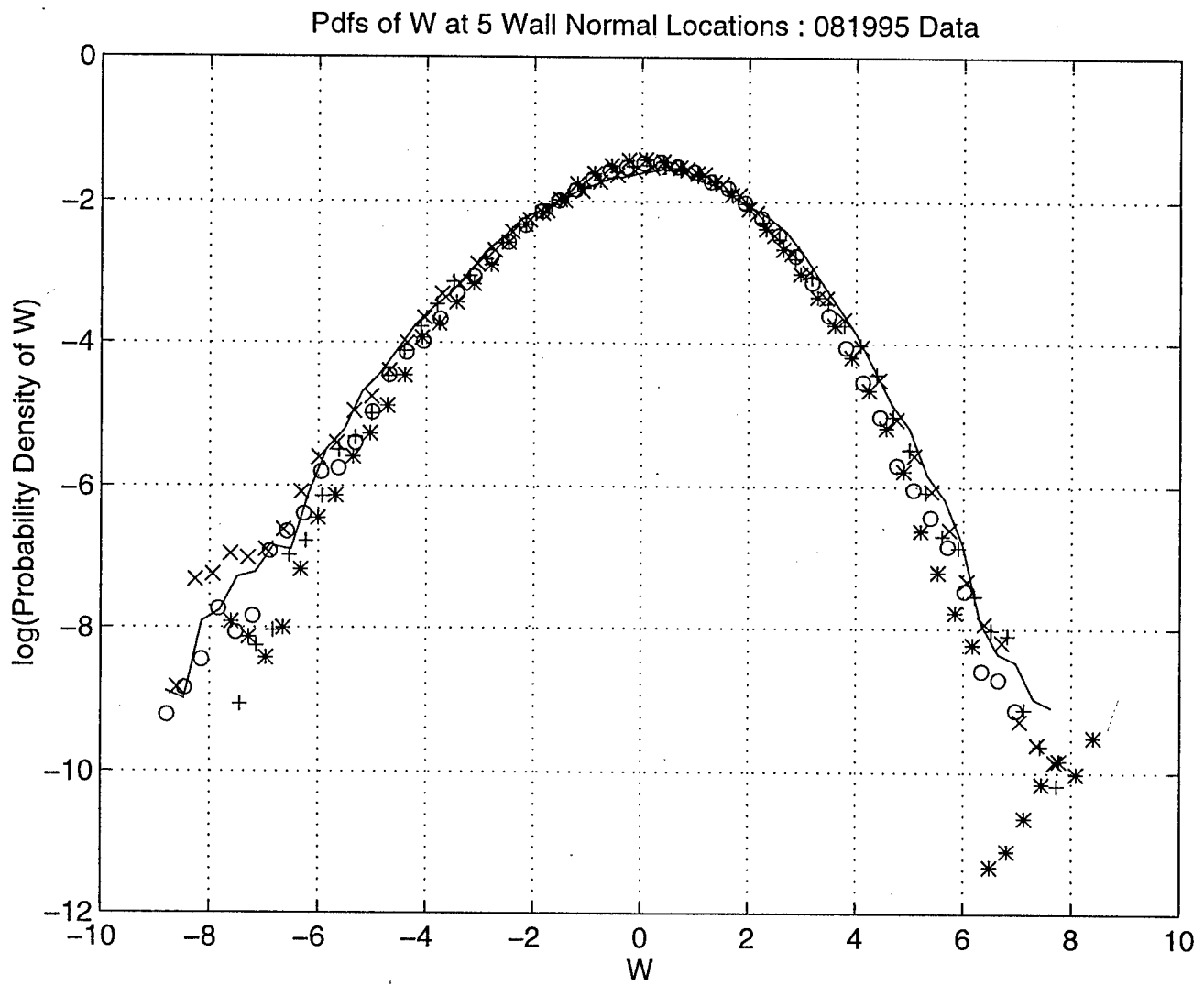


Figure 1



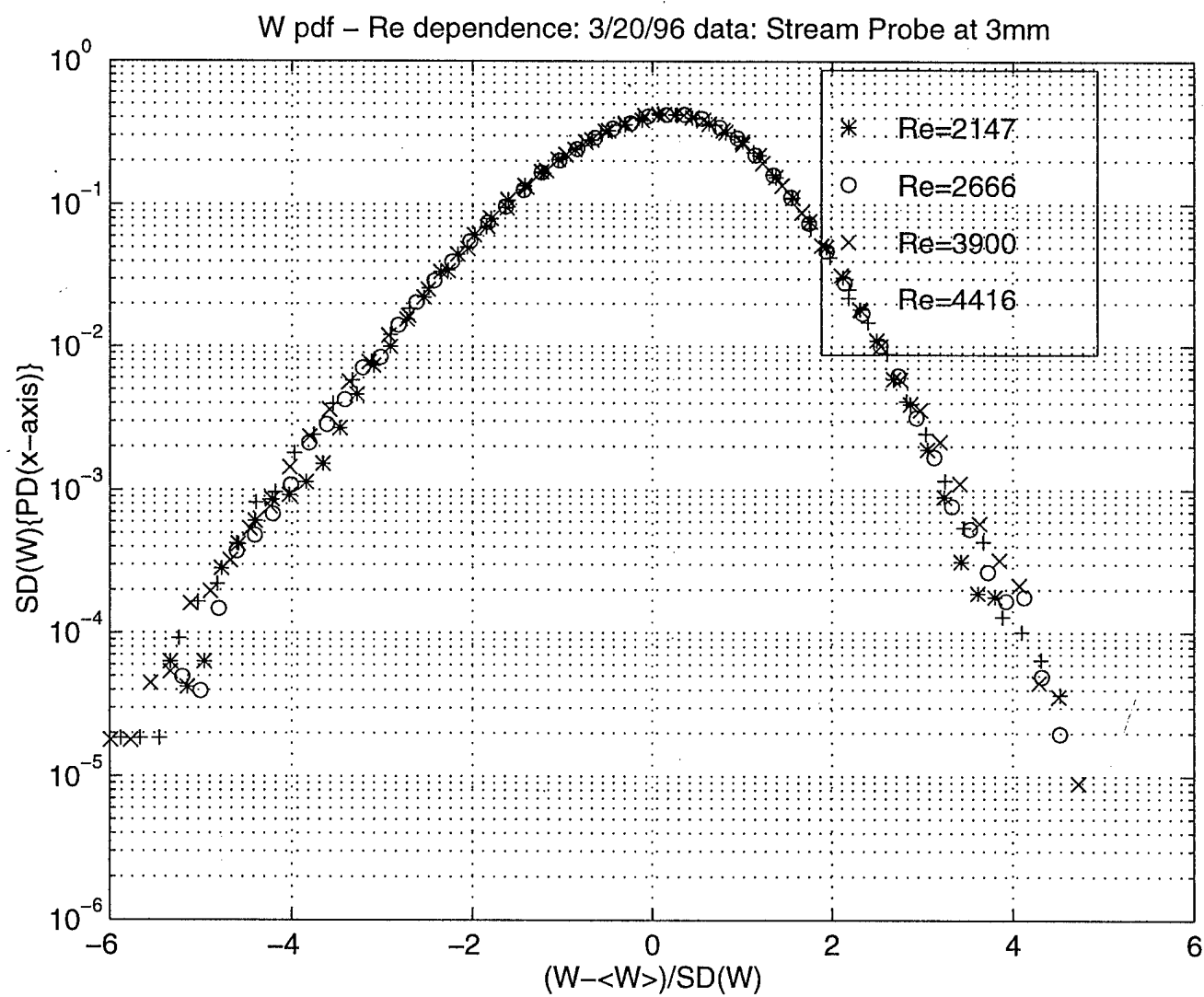


Figure 2

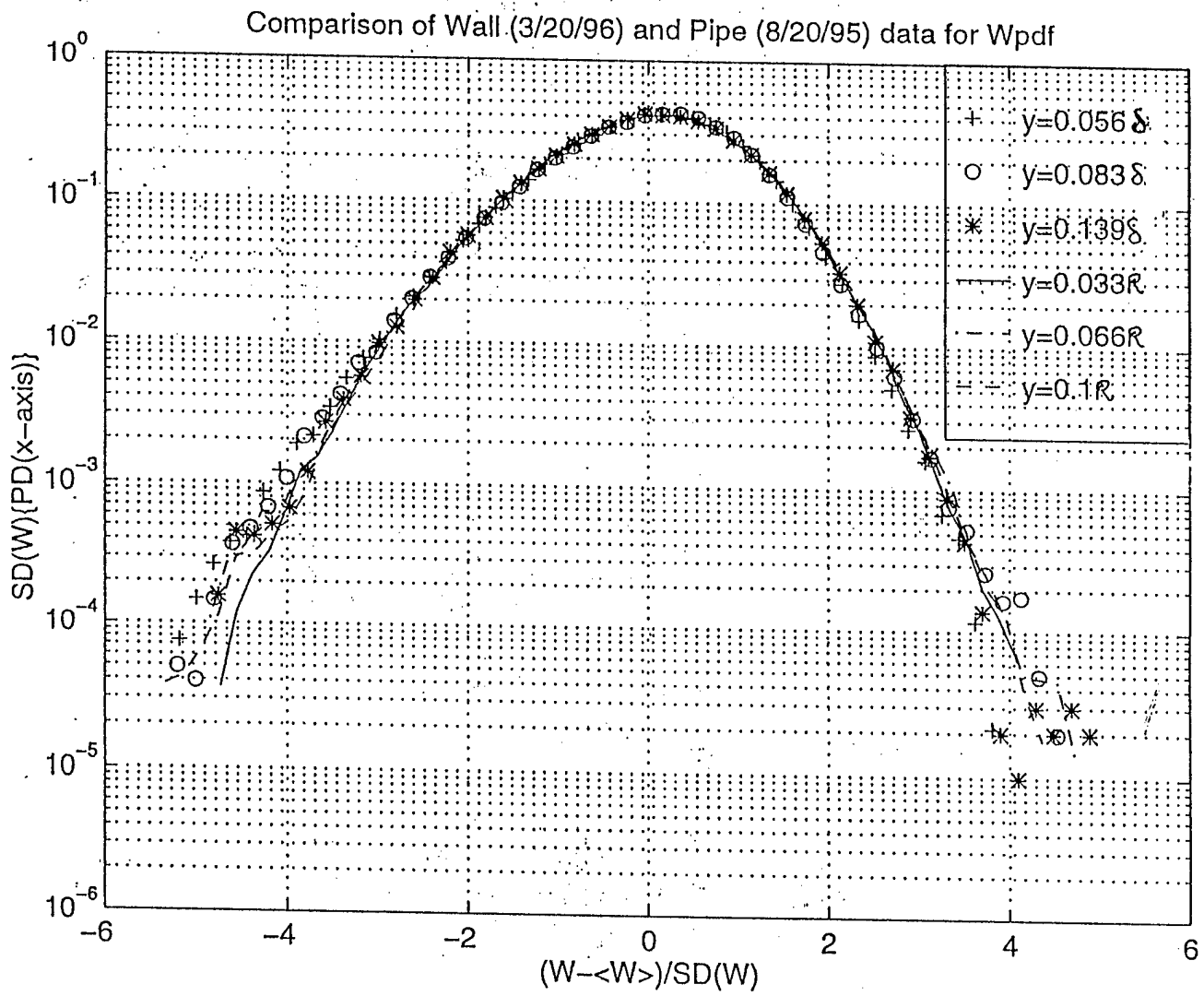


Figure 3